Plane Trigonometry

Terrametra Resources

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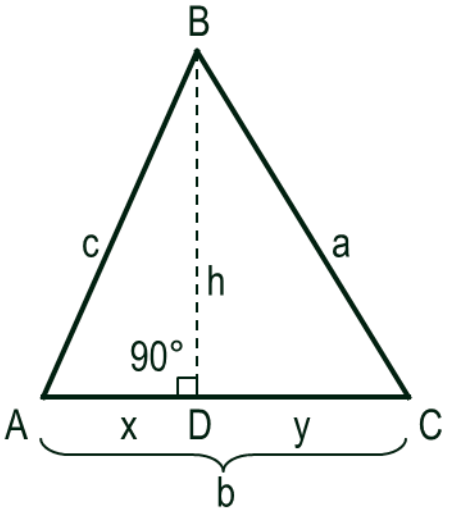
"LAW of SINES"

- **Equation 1** $h = c \cdot \sin A$
- **Equation 2** $h = a \cdot \sin C$



Equate the right sides of equations 1 and 2 and rearrange …

- **Equation 3** $\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$

Similarly: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$ and $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

"LAW of SINES"

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

… or …

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

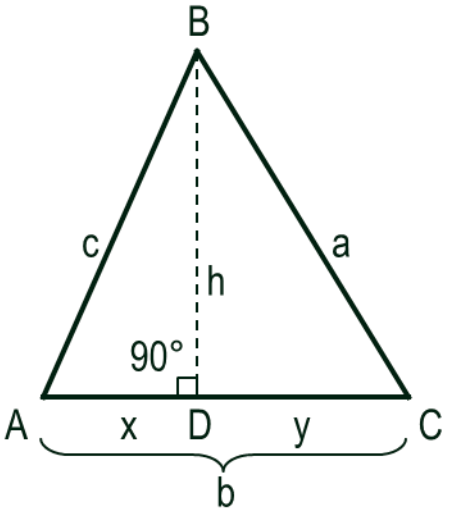
"LAW of COSINES"

- **Equation 1** $h^2 = c^2 - x^2$
- **Equation 2** $h^2 = a^2 - y^2 = a^2 - (b - x)^2$

Equate the right sides of equations 1 and 2 and rearrange …

$$a^2 - (b - x)^2 = c^2 - x^2$$

- **Equation 3** $a^2 = b^2 + c^2 - 2bx$
- **Equation 4** $x = c \cos A$



Substitute for x from equation 4 into equation 3…

"LAW of COSINES" $a^2 = b^2 + c^2 - 2bc \cos A$

Similarly:

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

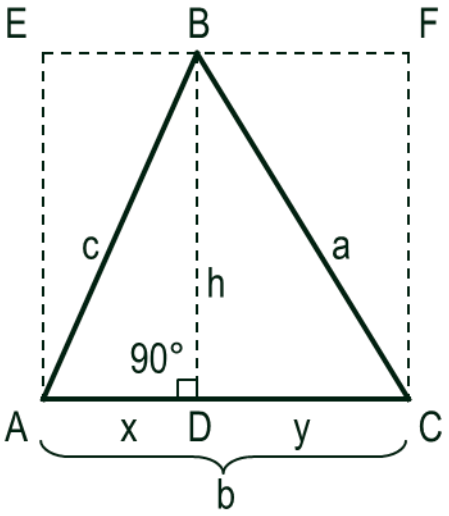
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Plane Triangle Areas

$$Area\ ABD = Area\ ABE = \frac{x \cdot h}{2}$$

$$Area\ CBD = Area\ CBF = \frac{y \cdot h}{2}$$



- **Area of Triangle ABC…** $Area = \frac{b \cdot h}{2}$

$$h = a\ \sin C$$

- **Area of Triangle ABC…** $Area = \frac{a \cdot b \cdot \sin C}{2}$

$$b = \frac{a \cdot \sin B}{\sin A}$$

- **Area of Triangle ABC…** $Area = \frac{a^2 \cdot \sin B \cdot \sin C}{2 \cdot \sin A}$